Channel Capacity Maximization in MIMO System via Periodic Circulant Discrete Noise Vector

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***Abstract*— Multiple-input multiple-output (MIMO) systems are today known as one of the most promising research areas of wireless communication. This paper proposes a MIMO systems capacity enhancement by using the convolution of the periodic Circulant Matrix-vector signal. Useful signals from Statistical dependence between discrete noise and transmitting signals adding to the receiver provide linear shifting of MIMO channel capacity to Electronics extent. In this paper, we examine the channel capacity, outage probability and SNR of MIMO receiver by adding a log determinant signal validated by numerical simulation.**

Keywords: MIMO, Periodic Circulant signal, Channel Capacity, Outage Probability

I. INTRODUCTION

Multiple-input multiple-output (MIMO) system has attracted significant interests among researchers and developers of new generation wireless systems because of its potentials of achieving high spectral efficiency, by multiplexing multiple users on the same time-frequency resources. MIMO system is successfully investigated and deployed where the rapidly increasing demand for multimedia challenges tremendously with high data rate and reliability. Using multiple antennas known as MIMO technology, at both the transmitter and receiver, results in a further increase in the capacity, provided that the environment is rich scattering. Besides that, the telecommunication industry put MIMO as a most promising research area of wireless communication, research of transmission is still facing path loss, shadowing and Fading.

MIMO system estimates channel capacity as a linear increase of spectral efficiency by utilizing diversity and spatial multiplexing techniques. The capacity of MIMO systems utilizes random matrix theory which depends on the number of transmitting (N_T) and receiving antenna (N_R) [1].

Additive White Gaussian Noise (AWGN) channel provides a safe signal from fading, amplitude loss, and phase distortion with flat frequency response and linear phase response.

The key advantage of the stationary random process is an Ergodic process which gives the independent mean value of the sequence. Infinite sequence average (ensemble average) is equal to the time average estimates single sequence which has the same average of infinite sequence that makes it possible to calculate any sample of that kind of the sequence [5].

However, recent works have proposed extracting useful signals using the Monte Carlo equation calculating the sum of probable bits signal as coefficient terms which are in scalar form. The application of Monte Carlo is very useful in today's applications of digital signal extracting on computational networks. From a random digital variable signal which has probability mass function is greater than zero can be extracted as an expected function by Monte Carlo equation and from the expected function can be generated a new signal with the same probability. Here, a discrete sum of function helps to generate an expected signal without restriction with equal probability [11].

Here, channel capacity enhancement based on a periodic circulant signal has been proposed as an appropriate solution for improving channel capacity in the MIMO system. We proposed, in the MIMO system by adding the non-negative Eigen value of periodic circulant signal with its log determinant on the receiver can improve its channel capacity.

In this paper, we add a periodic circulant signal in the receiver to improve the data rate of MIMO, using the Monte Carlo equation from statistical dependence transmitting and noise discrete signal, then calculate a set of expected values with its function [4]. The proposed concept helps to enhance the capacity of the MIMO system by recovering losses signals in the receiver side, Shown in a tree structure figure 1.

This paper is organized as follows; System model description is presented in section 2. Followed by useful periodic circulatant signal description with a mathematical model in section 3. Section 4 describes the proposed MIMO channel capacity mathematical model, simulation results comparing channel capacity rate between the existing and proposed system with SNR comparisons are described and the Outage probability with CDF comparison is described in section 5. Finally, paper conclusions are stated in section 6.

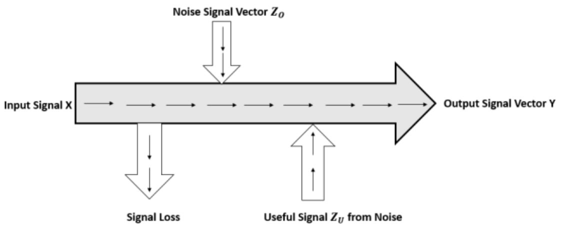

Fig.1. Tree structure of the MIMO channel capacity maximization

II. SYSTEM MODEL

In the MIMO system, varying PDF of transmits signal provides mutual information that can measure the information between two probabilities distributions [2] if the channel input and output are vector value instead of scalar once. And the AWGN channel removes endowing phenomena of Ergodic quantities that contain each codeword. Here, the definition of Entropy is also becoming the same with mutual information when channels input and output are vectors instead of scalar [2] and the Eigen values from samples of complex exponentials are an excellent feature of circulant matrix. This is the same fundamental Idea with a different expression used by Eigen values of matrix and transfer function of linear LTI systems [3]. In this paper, we calculate the MIMO channel capacity based on matrix theory [1].

The complexity of the system increases with the increasing super-polynomial value of codeword string n. The slice functions [12] are used for measuring the circuit sizes. $For\ n \geq 1 \qquad complexity\ factor = f_L{}^{(n)}$

If the value of n is sufficiently large then NP-complete language [12] estimates complex circuits.

$$f_L{}^{(n)} = \begin{cases} 1 & if\ codeword\ length = n \\ 0 & otherwise \end{cases}$$

The output of AWGN channel MIMO has coded sequence of transmitting signal and not effected by Fading, amplitude loss, and phase distortion problem. Although the considering signal will not suffer, so we choose a high probability non-confusable bit sequence from those set of the sequence [2]. In the MIMO system, by using a zero-mean Circular Symmetric Complex Gaussian (ZMCSCG) signal, we can estimate maximum channel capacity. So we assume all transmitting, receiving and noise vector are ZMCSCG.

In this system, we achieve maximum capacity when the receiver receives an auto-correlated signal from a transmitter which is a combination of the autocorrelation function of transmitting signal, receiving signal and useful log determinant signal. Here, Ergodic average provides the same PDF signals by comparing useful signals, we used the log determinant circuit because the signal is in random form. Then our proposed system model is shown in figure **2**.

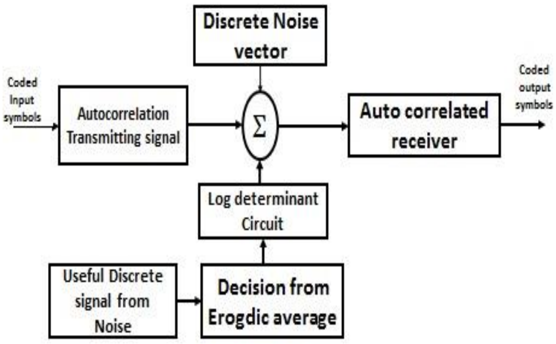

Fig.2. Maximization of MIMO channel Capacity using Noise vector

Here we assume discrete-time signal vectors are positive Hermitian, validating the function of a wide-sense stationary random process with circulant and good approximation [3]. The expression of circular convolution of discrete LTI system has a key feature which is the same as a multiplication between the circulant matrixes. Here, the circular convolution of the periodic circulant input signal is X, random matrix H with noise vector Z then discrete LTI system can be express in terms as [1] $\mathbf{Y} = \sqrt{\frac{E_X}{N_T}} Hx + Z_O$.

Now, we area adding useful circulant periodic signal ($\mathbf{Z_U}$), Taken from discrete noise vector ($\mathbf{Z_O}$). Then the receiving signal becomes

$$Y = \sqrt{\frac{E_X}{N_T}} Hx + Z_U + Z_O \tag{1}$$

Let us assuming signal transmitting on the AWGN channel of MIMO with noise has discrete i.i.d complex Gaussian sequence [10]. We assume here, the periodic circulant signal X and $\mathbf{Z_O}$ is in coded form and can be express as $\mathbf{X}(\mathbf{k}) = \sum_{\mathbf{l=0}}^{\infty} \mathbf{X}(\mathbf{k} + \mathbf{l}.\mathbf{N})$, with non-negative complex elements and noise vector $\mathbf{Z_O}(\mathbf{k}) = \sum_{\mathbf{l=0}}^{\infty} \mathbf{Z_O}(\mathbf{k} + \mathbf{l}.\mathbf{N})$, where the value of K and N is properly selected. A discrete LTI system utilizes random matrix theory as a form of convolution. The elements of the noise vector $\mathbf{Z_O}$can be written in matrix form

$$Z_O = \begin{bmatrix} Z_0 & Z_{(-1)} & Z_{(-2)} & \dots & \dots & Z_{(-N+1)} \\ Z_1 & Z_0 & Z_{(-1)} & \dots & \dots & Z_{(-N+2)} \\ \dots & \dots & Z_0 & \dots & \dots & \dots \\ Z_{N-2} & \dots & \dots & \dots & \dots & Z_1 \\ Z_{N-1} & Z_{N-2} & Z_{N-3} & \dots & \dots & Z_0 \end{bmatrix}$$

Here each element is coded and has equal power spectrum with plausibly asymptotically equal distribution from the left or right corners as

$$\mathbf{Z_{N-1}} \approx \mathbf{Z_{(-1)}} \text{ and } \mathbf{Z_{(-N+1)}} \approx \mathbf{Z_1}$$

And so on, Where the Maximum mutual information obtains by varying transmitting PDF signals in the receiver which estimates channel capacity of the system. It can be expressed as $\mathbf{I(X;Y) = H(Y) - H(Y|X)}$. We assume $\mathbf{Z}$ and $\mathbf{X}$ be statistically dependence discrete vector signal then, $\mathbf{H(X/Y) = H(Z) - H(X|Z)}$ and mutual information is taken as

$$\mathrm{I(X;Y) = H(Y) + H(Y|X) - H(Z)} \quad (2)$$

By taking mutual information of respected signal in terms of entropy we can get the following relationship

$$\mathrm{I\ (X;Y) = \log_2(\det(\pi e R_{YY})) + \log_2\left(\det\left(\pi e R_{\frac{X}{Z}}\right)\right) - \log_2(\det(\pi e R_Z))}$$

Which results in the mutual information,

$$I\ (X;Y) = log_2\left(det(R_{YY})\ (R_{\frac{X}{Z}})\ (R_Z{}^{-1})\right). \quad (3)$$

Currently Available Channel State Information **(CSI)**on the transmitter cannot improve the channel capacity of MIMO when SNR is high [1]. Here, we assume **CSI** is not available and **H**is also unknown to transmitter then the energy is equally spreading in all direction and then autocorrelation function of the transmitting signal vector X becomes,

$$R_{XX} = I_{N_T} \quad (4)$$

We expect that we can receive a maximum amount of mutual information only when the receiving signal has also autocorrelation then,

$$\mathrm{R_{YY} = E((\sqrt{\tfrac{E_X}{N_T}} Hx + Z_U + Z_O)(\sqrt{\tfrac{E_X}{N_T}} Hx + Z_U + Z_O)^H)}$$

$$= \mathrm{\frac{E_X}{N_T}\ (E(Hxx^H H^H + Z_U Z_U{}^H + Z_O Z_O{}^H))}$$

To make simpler, we assume independently and transmitting energy equals to 1. which leads us,

$$\mathrm{R_{YY} = \frac{E_X}{N_T}\ ((R_E + R_U + R_O)}$$

After that we can denote useful signal as $\mathrm{R_{X/Z} = Z_U Z_U{}^H = R_U \& R_Z = Z_O Z_O{}^H = R_O}$

Then, the resulting mutual Information can be written as,

$$I\ (X;Y) = log_2\left(det\left(\frac{E_X}{N_T}((R_E + R_U + R_O))(R_U)(R_O{}^{-1})\right)\right) \quad (5)$$

III. Useful Periodic Circulant Signal

In this section, we focus on the useful periodic circulant signals. The useful noise vector is linear dependence on the eigenvectors of the Hermitian matrix of orthonormal then the noise vector can be represented as squared Frobenius norm of MIMO channel. Here, the transmitting power of each antenna is equal to 1. The useful signals Z_U is calculated from the noise vector as a statically dependence with transmitting signal vector, which is calculated as Monte Carlo equation from a set of the expected value of the function discrete noise signal [4]. Depending upon X and $\mathbf{Z_O}$**,** our expected signal becomes ZMCSCG, circulant and periodic. The useful signal calculated from Monte Carlo equation is

$$\mathbf{E}(Z_U(Z_O)) = \sum\nolimits_{m\epsilon Z} Z_U\, f_{Z_O}(m) \quad (6)$$

Where **m** is our expected value from noise through using the function $\mathbf{f_{Z_O}}$ and the element of the circulant periodic matrix. By comparing useful signal with the ergodic average, the decision circuit provides similar coded useful elements matrix signal $\mathbf{Z_U(k) = \sum_{l=0}^{\infty} Z_O(k + l.N)}$, which gives similar value coded elements of **X**. The elements of signal make periodic signal which requires additional hardware for the same codeword circuit, although it is not complex due to its same polynomial order. Then the useful matrix is,

$$\mathrm{Z_U} = \begin{bmatrix} Z_{U_0} & Z_{U_{-1}} & \dots & Z_{U_{(-m+1)}} \\ Z_{U_1} & \dots & \dots & Z_{U_{(-m+2)}} \\ \dots & \dots & \dots & \dots \\ Z_{U_{m-1}} & \dots & Z_{U_1} & Z_{U_0} \end{bmatrix}$$

Here, the diagonal elements provide its eigenvalues which have equal probability.

$$R_{X/Z} = E(Z_U Z_U{}^H) = \mathbf{E}\textstyle\sum_{l=1}^{M} \gamma_i = \sum_{l=1}^{M} \gamma_i \quad (7)$$

Assuming that the PDF of useful signals follows chi-square distribution [7]. This gives better performance when M tends to infinity. Then, the probability of useful signal calculated as

$$(\gamma_M) = \frac{1}{p}\sum_{m\epsilon Z} q(m)P = \frac{1}{P}\ E(q(Z_U)) \quad (8)$$

Where $\mathbf{Z_U}$ is the random variables that take the expected value of Z and q is sum of the function over countable values. The matrix-vector $\mathbf{Z_U}$ is circulant and periodic so we can solve the matrix similar to the covariance matrix of both signal, divided by N_{Min}. The value of $\boldsymbol{\gamma}$m is taken as a log basis due to the random nature of the useful signal.

$$\gamma_m = log_2(det(real(Z_U))) \qquad (9)$$

Useful Co-variance receiving matrix gives the value of $\mathbf{Z_U}$ as

$Z_U = \frac{E_X}{N_M} I_M$, in case oftransmitting orthonormal signals, $\mathbf{Z_U}$becomes$Z_U = \frac{1}{N_M} I_M$then the resulting equation (**8**)is

$$\gamma_M = \frac{1}{N_M} log_2(det(real(I_M))) \qquad (10)$$

IV. THE MIMO CHANNEL CAPACITY

The ergodic[2] process is the most convenient way to express the existing MIMO channel capacity which is calculated as statistical notation. The channel capacity of the existing MIMO [1] is:]

$$(C) = {}_{Tr(Rxx)=N_T}^{Max} log_2 \left(det \left(\begin{matrix} I_{N_R} + \\ + \frac{E_X}{N_T N_O} HH^H \end{matrix} \right) \right) \qquad (11)$$

Since R_O is a complex circulant periodic sequence, it gives Eigen values when it multiplies by its inverse. By assigning all found values in an equation provides new improved mutual information of the MIMO system.

$$I(X;Y) = log_2 \left(det \left(\frac{E_X}{N_T} \left(\left(R_E R_U R_O^{-1} + R_U{}^2 R_O{}^{-1} + I_{N_R} R_U \right) \right) \right) \right) \qquad (12)$$

Then the resulting useful circulant signal has non-negative values that work as an increasing factor. The value of R_U is scalar and can be replaced byγ_M. Then the final mutual information be,

$$I(X;Y) = log_2(det \left(I_{N_R} \gamma_M + \frac{E_X}{N_T N_O} \gamma_M{}^2 + \frac{E_X}{N_T N_O} \gamma_M HH^H \right) \qquad (13)$$

Hence, the calculated channel capacity of the MIMO from mutual information is

$$channel\ capacity\ (C) = {}_{Tr(Rxx)=NT}^{Max} log_2 \left(det \left(\begin{matrix} I_{N_R} \gamma_M + \frac{E_X}{N_T N_O} \gamma_M{}^2 \\ + \frac{E_X}{N_T N_O} \gamma_M HH^H \end{matrix} \right) \right). \qquad (14)$$

This channel capacity of the system depends upon non zero, non-negative real value of γ_M, these values give the overall MIMO system to some additional computation. Even by providing a proper selection of **n** create some complexity due to the requirement of more super-polynomial circuit design in the receiver side which gives bulk size estimation than the existing system.

V. SIMULATION RESULTS

To verify the theoretical model proposed in section 3 and section 4, MATLAB implementation is employed to showing capacity rate and obtains estimates of the source and mixing discrete signal through a matrix of N_T and N_R followed the algorithm of Channel capacity [1]. However, adding a useful logarithmic determinant on the receiver estimate a higher capacity rate. Simulation results show that linearly increasing in channel capacity is a similar way of increasing in value of N_T and N_R which also helps to reduce the necessity of requiring high channel rank for high spectral efficiency. In MIMO system channel capacity is increased with increase in N_T and $N_R$values however, the addition of the determinant signal on receiver provides linear shifting of capacity rate to some positive extent as shown in figure **3.**The Simulation results show the shifting value is linearly increasing with a number of increased transmitting and receiving antennas. N_T has a greater impact on achieving a new capacity rate on a receiver. Statistical notation of CDF arises floating-point round error, to recover this error we take convenient arbitrary number known as binning technique. The comparison between the existing and proposed MIMO channel capacity rate using MATLAB is clearly shown in figure 3.

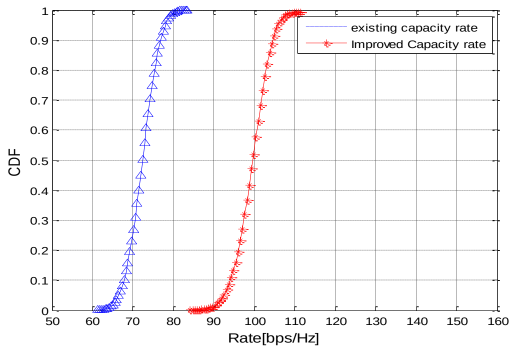

Fig.3. Comparison of MIMO channel Capacity rate between the existing and proposed system

Bit-Error-Rate (**BER**) and Signal-to-Noise Ratio (**SNR**) of the receiving signal have an inverse relation. An error occurs on transmission in a fixed interval of studied time which is known as BER. SNR is the ratio of useful signal and noise signal as a form of voltage or power. The SNR formula relating with BER in terms of diversity is $\mathbf{BER} \propto \frac{\mathbf{1}}{\mathbf{SNR^d}}$where **d** represents the order of diversity of the MIMO system which provides better reliability of the system. But on the other hand spatial multiplexing of MIMO uses different antennas with a differential stream which estimates a high data rate. Here, the proposed model has linearly increasing SNR with high value, then the existing MIMO system model estimates low BER shown in figure **4**.

As we know that, there is no possibility of completely removing decoding error however it can be minimizing to some level by using an advanced decoding scheme, in compare with a certain threshold value. If the random error bit rate is greater than the threshold value, the corresponding probability is called an outage probability. Here, the mutual information and outage probability can be related as [9].

$$P_{out} = P(I < R). \quad (15)$$

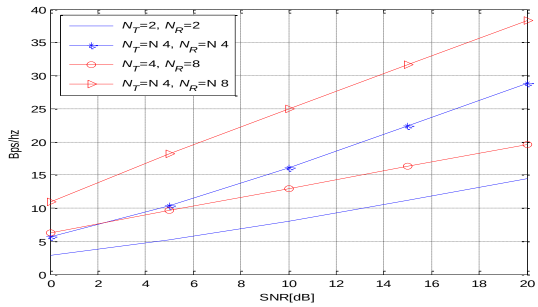

Fig.4. Comparison of SNR between existing and proposed Model

Flat fading estimates capacity is random instantaneous value and taking constantly for a coded block of information. Using the probability law, the outage probability of capacity in terms of total probability is [9].

$$P = P(I < R) = \sum_C P[I < R|C]P[C] \quad (16)$$

And its complex exponential outage probability is calculating statically as Monte Carlo estimation. When **BER** and signal-to-Noise-interference ratio (**SINR**) are equals, the threshold value depends upon the detection and diversity order of MIMO.

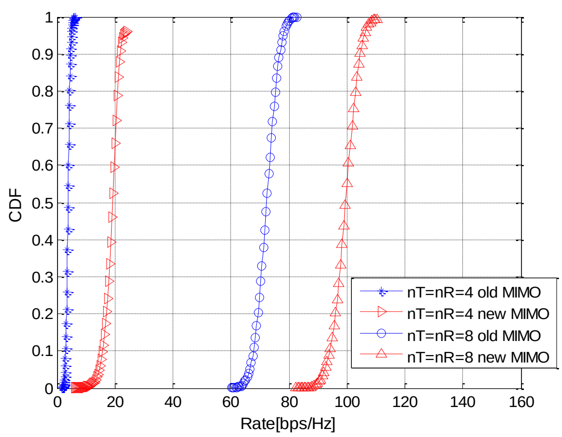

Fig.5. CDF comparison between Existing and proposed MIMO system

The outage probability has an inverse relation with SNR. Here, we can be optimized the transmitting signal by reducing uncertainty in channel distribution. Simulation results figure **5** shows the higher value of **M** provides good CDF of outage probability. So from our simulation, we found that the statistical bins values for calculating CDF are directly affecting the capacity rate of the MIMO system.

VI. Conclusion

We proposed a MIMO systems capacity enhancement by using the convolution of the periodic circulant Matrix-vector signal. The expression we derived from the statistical dependence of discrete transmitting and noise signal with the AWGN channel is beneficial to carry information of MIMO at a higher rate than the existing system with the requirement of a higher number of transmitting and receiving antennas. The linear shifting of capacity mainly depends on the higher value ofN_T in arandom matrix. The higher SNR with higher channel capacity has been obtained due to the interception of digital communication signals. Our model predicted Proper selection of transmitting signal code word could reduce the complexity of the system however proposed MIMO system is quite complex than existing once. Our result shows, by using the proposed system we can increase channel capacity in the MIMO system.

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